A POSITIVE SEIBERG-WITTEN SPINOR IS L^∞-BOUNDED

CELSO M. DORIA
UFSC - DEPTO. DE MATEMÁTICA.

ABSTRACT. It is shown that a positive spinor (1.2.2) obtained as a solution to the non-homogeneous 2^{nd}-order Seiberg-Witten equation is L^∞-bounded. This is a generalization of the basic lemma, which we name $\mathcal{SW}_\alpha$-lemma, proved for the solutions of the homogeneous equation ([2]) and which consequences are fundamental to the Seiberg-Witten theory. The $\mathcal{SW}_\alpha$-lemma is applied in [4] to the study of the boundary value problems associated to the $\mathcal{SW}_\alpha$-equation.

1. Introduction

Let X be a compact smooth 4-manifold without boundary. In our context, the Seiberg-Witten equations are the 2^{nd}-order PDE obtained as the Euler-Lagrange equation of the functional defined in (1), which analytical aspects were first studied in [8] and the topological ones in [3].

1.1. Spin^c Structure. The space of $Spin^c$ structures on X is identified with

$$Spin^c(X) = \{\alpha + \beta \in H^2(X), \mathbb{Z}) \oplus H^1(X, \mathbb{Z}_2) \mid w_2(X) = \alpha (mod\ \ 2)\}.$$

For each $\alpha \in Spin^c(X)$ we associate a pair of bundles

$$\alpha \in Spin^c(X) \quad \rightsquigarrow \quad (\mathcal{L}_\alpha, \mathcal{S}_\alpha{}^+).$$

From now on, we considered fixed on X a Riemannian metric g and on $\mathcal{S}_\alpha$ an hermitian structure h.

Let P_α be the U_1-principal bundle over X obtained as the frame bundle of $\mathcal{L}_\alpha$ ($c_1(P_\alpha) = \alpha$). Also, we consider the adjoint bundles

$$Ad(U_1) = P_{U_1} \times_{Ad} U_1 \quad ad(\mathfrak{u}_1) = P_{U_1} \times_{ad} \mathfrak{u}_1,$$

where $Ad(U_1)$ is a fiber bundle with fiber U_1, and $ad(\mathfrak{u}_1)$ is a vector bundle with fiber isomorphic to the Lie Algebra $\mathfrak{u}_1$.

1.2. $\mathcal{SW}_\alpha$-Functional. Let $\mathcal{A}_\alpha$ be the space of connections (covariant derivative) on $\mathcal{L}_\alpha$, $\Gamma(\mathcal{S}_\alpha^+)$ the space of sections of $\mathcal{S}_\alpha^+$ and $\mathcal{G}_\alpha = \Gamma(Ad(U_1))$ the gauge group acting on $\mathcal{A}_\alpha \times \Gamma(\mathcal{S}_\alpha^+)$ as follows:

$$g.(A, \phi) = (A + g^{-1}dg, g^{-1}\phi).$$

$\mathcal{A}_\alpha$ is an afim space which vector space structure is isomorphic to the space $\Omega^1(ad(\mathfrak{u}_1))$ of $ad(\mathfrak{u}_1)$-valued 1-forms. Once a connection $\nabla^0 \in \mathcal{A}_\alpha$ is fixed, a bijection $\mathcal{A}_\alpha \to \Omega^1(ad(\mathfrak{u}_1))$ is explicited by $\nabla^A = \nabla^0 + \mathbf{A}, A \in \Omega^1(ad(\mathfrak{u}_1))$. $\mathcal{G}_\alpha = Map(X, U_1)$,

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Campinas, 19 a 22 de novembro de 2003